



[6, 7] in order to estimate the additional copper losses.

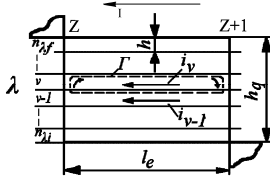


Fig. 3 An elementary section of layer  $\lambda$

### III. THE SLOT MAGNETIC FIELD

In the slot of an electric machine there are two kinds of magnetic fields. The two fields can generate additional losses in the copper conductors located in the slot.

Synchronous generators are generally high and very high power electric machines, with windings made of copper bars located in slots. If an alternating current flows through such bar, it generates a leakage field with lines closing transversely through the slot (Fig. 4).

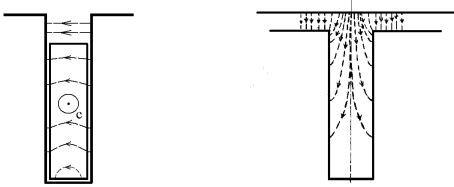


Fig. 4 The slot magnetic fields:  
a) of inner current; b) and of external current

This leakage field, time-variable, induces in the bar eddy currents which determine an unevenly current distribution on the bar surface. Actually, it is found that the current density is lower close to the slot base (Fig. 5) and higher close to the slot opening (towards the air gap).

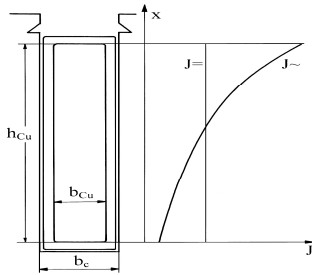


Fig. 5 Current density distribution in the bar

If three or many bar layers are in the slot, then similar distribution curves are got.

Because of this effect, the electrical resistance of the bars in alternating current ( $R_a$ ) is several times higher than the electric resistance of the same bar in direct current ( $R_0$ ):

$$R_a = k_{ra} \cdot R_0 \quad \text{where } k_{ra} > 1 \quad (1)$$

Consequently, in the alternating current generator winding, additional losses appear unavoidably.

For the computation of energetic parameters the lines of the leakage field from the slot are considered to be parallel to the slot base, i.e. perpendicular to the slot walls.

This field model is the basis for the computation of the current displacement effect, namely for the coefficient of

increase of the bar electric resistance in alternating current ( $k_{ra}$ ).

A slot where  $m$ -layers of rectangular bars are located is considered (Fig. 6).

Each layer has certain coefficient of electric resistance increase. By example, for the bar from the layer "p", the coefficient of resistance increase is [8]:

$$k_p = \varphi(\xi) + \frac{I_u (I_u + I_p)}{I_p^2} \Psi(\xi) \quad (2)$$

where the current  $I_p$  is the current from the considered layer, and  $I_u$  is the sum of the currents from all the layers located below the considered layer.

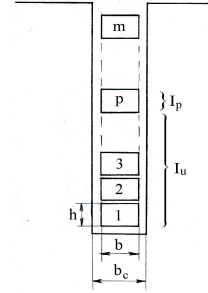


Fig. 6 Open slot with „M”- bars

If the same current flows through all the  $m$  – bars, an average value per slot of the coefficient (2) results [8]:

$$k_{ra} = \varphi(\xi) + \frac{m^2 - 1}{3} \Psi(\xi) \quad (3)$$

where the functions  $\varphi(\xi)$  and  $\Psi(\xi)$  have the well know expressions [8].

The problem of current displacement effect could be solved in this way also in the case when the bar is formed by many subconductors, connected in parallel.

The excitation field of a synchronous generator is generated by the current from the excitation winding, generally placed on the generator rotor. This magnetic field passes from rotor through air-gap into stator teeth, but enters also into the slot space, where the bars of stator winding are located (Fig. 7). It generates inner circulating currents through the bar subconductors, which are short-circuited at the ends, forming short-circuit loops. These currents cause additional losses in the stator winding, leading to the decrease of the electric energy conversion efficiency.

This magnetic field from the stator slot area was calculated, with view to determining the currents and additional losses in bars. For this purpose, in a previous paper [9] the conformal mapping method was used, in order to benefit from the advantages of the analytical methods for field computation.

The main results of the paper [9] in this paragraph are presented.

It is considered the numerical example in which the slot width  $b_c = 24$  mm, the air-gap  $\delta = 5$  mm and flux density  $B_{\delta \max} = 1$  T.

The magnetic field in the slot was calculated by analytical method (conformal mapping method) and by finite element method (FEM), at different depths ( $y = 0 \dots -10$  mm). Some computation results are presented in Fig. 8. Due to the

symmetry of the magnetic field in the slot, the distribution curves of the magnetic field radial components were represented only on a half of the slot width (from the slot wall up to the slot axis)..

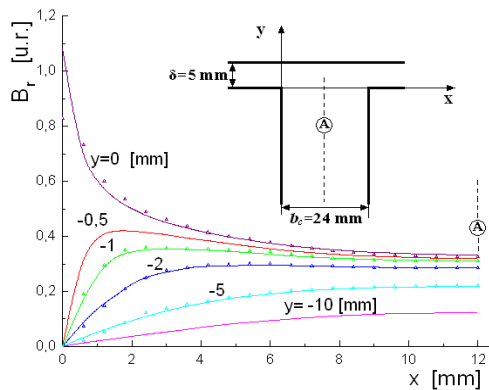


Fig. 8 Some curves of the flux density radial component across slot width direction: analytical method

On the basis of many computation examples, the following analytical relations of the radial magnetic field distribution in the slot width direction are proposed:

$$\begin{aligned} B_r(x) &= B_{rm} \sin\left(\frac{x}{b_c} \pi\right), \text{ for } |y| \geq 2\delta \\ B_r(x) &= B_{rm} \left(\sin\left(\frac{x}{b_c} \pi\right)\right)^{0.8} \text{ for } |y| \approx \frac{3}{2} \delta \\ B_r(x) &= B_{rm} \left(\sin\left(\frac{x}{b_c} \pi\right)\right)^{0.5}, \text{ for } |y| \approx \delta. \end{aligned} \quad (4)$$

Close to the air gap, when  $|y| < \delta$ , the distribution can be no more approximated by a sine curve; within these zones, the curves are like those ones calculated in Fig. 8 for  $y = -2$  mm,  $y = -1$  mm,  $y = -0.5$  mm.

In order to develop as general as possible analytical solutions, the radial component of the magnetic flux density ( $B_{rm}$ ) in the slot axis and its distribution in the depth direction was analyzed further on.

By using many families of curves got by computation, also the MATHLAB facilities, an analytical relation for approximating the values of the magnetic field density in the slot axis was built under the following form, valid in any point from the slot axis:

$$B_{rm} = e^{-\alpha} B_{\delta \max} \quad (5)$$

where:

$$\alpha = m_m \left| \frac{y}{\delta} \right| + n_n \quad (6)$$

and:

$$m_m = \frac{38,39 \left(\frac{b_c}{\delta}\right)^3 - 665,1 \left(\frac{b_c}{\delta}\right)^2 + 6507 \left(\frac{b_c}{\delta}\right) + 1061}{\left(\frac{b_c}{\delta}\right)^3 + 909,4 \left(\frac{b_c}{\delta}\right)^2 + 2654 \left(\frac{b_c}{\delta}\right) - 1246}$$

$$n_n = 0,0857 \left(\frac{b_c}{\delta}\right)^1 + 0,3786.$$

By using the approximate relation (5) the value of induction in the slot axis can be determined, avoiding the difficulties in using the conformal mapping relations. The relations (4) and (5) in the computing program for additional losses estimation was included.

#### IV. THE CALCULUS OF THE CURRENTS

One considers the elementary section belonging to the layer  $\lambda$  (Fig. 3). One chooses a  $\Gamma$ -curve traced through the upper parts of two consecutive strips with the order numbers  $v-1$  and  $v$ . One performs the line integral along this curve of electric field strength and obtains a relation between the currents of the two considered strips. The currents through the strips  $v$  and  $v-1$  are  $i_v$  and  $i_{v-1}$  respectively. The currents vary in a sinusoidal shape in time. In consequence, one can use complex quantities. One can calculate the  $\underline{L}_{cv}$  current of the  $v$  strip depending on the currents of the elementary conductors under the form:

$$\underline{L}_{cv} = \underline{S}_v \underline{L}_{c\lambda} + \underline{T}_v \underline{L}_{i\lambda} \quad (7)$$

Here  $\underline{L}_{c\lambda}$  is the sum of the two currents in the neighbor conductors in the same layer  $\lambda$ , and  $\underline{L}_{i\lambda}$  is the sum of all the currents in the conductors placed between the slot base and the  $\lambda$ -layer. For  $\underline{S}_v$  and  $\underline{T}_v$  one considers the following system of relations:

$$\begin{aligned} K_v &= \frac{b_{cu,v}}{b_{cu,v-1}}; \\ \underline{B}_v &= j \frac{2\omega\mu h^2 b_{cu,v}}{\rho_{cu} b_c}; \\ \underline{C}_1 &= 1 \underline{E}_1 = 0; \\ \underline{A}_v &= \frac{3K_v}{3 - \underline{B}_v}; \\ \underline{D}_v &= -\frac{3\underline{B}_v}{3 - \underline{B}_v}; \\ \underline{C}_v &= (\underline{A}_v - \underline{D}_v) \underline{C}_{v-1} - \underline{D}_v \sum_{\varepsilon=n_{\lambda_i}}^{v-2} \underline{C}_\varepsilon \\ \underline{E}_v &= (\underline{A}_v - \underline{D}_v) \underline{E}_{v-1} - \underline{D}_v \left( \sum_{\varepsilon=n_{\lambda_i}}^{v-2} \underline{E}_\varepsilon - 1 \right) \\ \underline{C}_{\Sigma f} &= \sum_{v=1}^N \underline{C}_v; \\ \underline{E}_{\Sigma f} &= \sum_{v=1}^N \underline{E}_v \\ \underline{S}_v &= \frac{\underline{C}_v}{\underline{C}_{\Sigma f}}; \\ \underline{T}_v &= \underline{S}_v \underline{E}_{\Sigma f} - \underline{E}_v. \end{aligned} \quad (8)$$

In these relations:  $\mu$  is the magnetic permeability of the conductor material (virtually equal with the magnetic permeability of the vacuum environment –  $\mu_0$ );  $b_{cu,v}$  – the width of the conducting cross section of an elementary conductor in the  $v$  strip;  $b_c$  – the width of the rectangular slot;  $j$  – is the imaginary unity.

The bar ends are treated similarly as the slot portions.

One then chooses another closed curve  $\Gamma_1$  traced through the first strips of the two successive subconductors  $\xi$  and  $\xi+1$ . This curve is closed at the two bar ends, where all the  $2m$ -subconductors are short-circuited. A line integral of the electric field intensity along  $\Gamma_1$ -curve was calculated, by means of which a relation between the currents of the subconductors was established. Thus are obtained  $(2m-1)$  relations between the strand currents. One also takes into account the fact that the sum of all the strand currents is always equal with the current of the bar. Consequently:

$$\sum_{\varepsilon=1}^{2m-1} G(\varepsilon, \xi) I_{\varepsilon} = \underline{U}_{erc}(\xi), \quad \xi=1, 2, \dots, 2m-1 \quad (9)$$

$$\sum_{\varepsilon=1}^{2m} I_{\varepsilon} = I$$

The  $G$  coefficients can all be calculated depending on geometrical dimensions and of material constants, all known, and  $\underline{U}$  is the electromotive force from the strand conductor induced by the radial magnetic fields. The system contains  $(2m)$  equations, with  $(2m)$  unknown variables and therefore the currents in the strand conductors can be determined.

Now one can calculate the currents of every strip for any strand conductor and one obtain for the  $v$  strip of the elementary conductor  $\xi$ :

$$\begin{aligned} \underline{I}_{v, \xi} &= \frac{P_{rv}}{P_{r \Sigma N}} \underline{I}_{\xi} + \\ &+ \frac{B_1}{2} \left[ \left( \underline{U}_v - \frac{P_{rv}}{P_{r \Sigma N}} \underline{U}_{\Sigma N} \right) \underline{I}_{c\lambda} + \right. \\ &\left. + \left( \underline{V}_v - \frac{P_{rv}}{P_{r \Sigma N}} \underline{V}_{\Sigma N} \right) \underline{I}_{u\lambda} \right], \end{aligned} \quad (10)$$

where, generally  $K_v = b_{cu,v} / b_{cu,v-1}$ . At hollow strand conductors of rectangular form there are only two dimensions ( $b_{cu1}$  and  $b_{cu2}$ ). Considering  $K = b_{cu2} / b_{cu1}$  the following relations for the coefficients are obtained:

$$\begin{aligned} P_{rv} &= K_1 \cdot K_2 \cdot \dots \cdot K_v ; \\ P_{r \Sigma N} &= \sum_{v=1}^N P_{rv} \\ \underline{S}_{\Sigma v} &= \sum_{\varepsilon=1}^v \underline{S}_{\Sigma \varepsilon} ; \\ \underline{T}_{\Sigma v} &= \sum_{\varepsilon=1}^v \underline{T}_{\Sigma \varepsilon} \\ \underline{S}_{\Sigma \Sigma v} &= \sum_{\varepsilon=1}^v \underline{S}_{\Sigma \Sigma \varepsilon} ; \\ \underline{T}_{\Sigma \Sigma v} &= \sum_{\varepsilon=1}^v \underline{T}_{\Sigma \Sigma \varepsilon} \\ \underline{S}_{\Sigma \Sigma \Sigma v} &= \sum_{\varepsilon=1}^v \underline{S}_{\Sigma \Sigma \Sigma \varepsilon} ; \\ \underline{T}_{\Sigma \Sigma \Sigma v} &= \sum_{\varepsilon=1}^v \underline{T}_{\Sigma \Sigma \Sigma \varepsilon} \end{aligned} \quad (11)$$

$$\begin{aligned} \underline{U}_v &= K \left( \underline{S}_{\Sigma \Sigma v} - \frac{2}{3} \underline{S}_{\Sigma v} \right) \\ \underline{V}_v &= K \left( v - 1 + \underline{T}_{\Sigma \Sigma v} - \frac{2}{3} \underline{T}_{\Sigma v} \right) \\ \underline{U}_{\Sigma N} &= \sum_{\varepsilon=2}^N \underline{U}_{\varepsilon} ; \\ \underline{V}_{\Sigma N} &= \sum_{\varepsilon=2}^N \underline{V}_{\varepsilon} \end{aligned}$$

and in a similar way for the ends winding.

Considering a constant current density on the cross section of the strip, one determines the losses in the strip. Adding the losses from all the strips, for all the elementary sections, one obtains the losses in the entire bar.

## V. CONCLUSION

Based on the previously presented model, using such kind of relations, a computing program for both currents and additional losses estimations was developed. The influence of any input data (strands number, transposition angle, geometrical dimensions, material data) can be considered. The program allows a faster analysis of a bar structure and, if is necessary, can provide an optimal bar structure. The research work will be continued with an experimental model in order to validate this approach.

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